Cycles in Rainfall and Validity in Prediction of Rainfall in Hawaii

CHESTER K. WENTWORTH[1]

IN THE PRESENT PAPER it is proposed first to discuss a recent book titled *Correlation of Cycles in Weather, Solar Activity, Geomagnetic Values, and Planetary Configurations* (Johnson, 1946). This discussion is followed by an application of a method of analysis therein described to the 56-year series of rainfall data for the Honolulu intake area known as the Honolulu Rainfall Index (Board of Water Supply, 1947: 180). The conclusion is reached that this method does not result in prediction of the rainfall for a future year with sufficient accuracy to be of practical utility.

The treatment by Johnson represents an enormous amount of labor in computing and compiling data. It carries much food for thought and exemplifies methods that other investigators will find useful for application to specific problems. The present discussion deals chiefly with the question of whether the data presented show the capacity of the method to predict future rainfall quantities with useful accuracy. Some attention is given to the general method and the suggested correlations between various other physical phenomena, but the present writer does not claim competent knowledge in most of these fields.

The method used by Johnson in the analysis of cycles, or periodicities, is that previously used by Dinsmore Alter and described by him in 1937 and in several other papers cited by Johnson. This procedure is comparatively simple and has doubtless been devised and used by various students having occasion to do such work. It consists essentially of a process of finding that constant interval between pairs of years such that the average difference in rainfall between the two members of a pair is a minimum. For any given interval, the average error or difference between the first year's rainfall and the next year's rainfall is called the A index, presumably meaning the "Alter index." The interval that is found to give the lowest A index is the period that is assumed to give the most reliable estimate of future values.

Johnson has carried the method much beyond this point by subjecting the A differences based on any one period to similar analysis to find a second period, and so on to several periods of diminishing importance. He has also made some use of an index he calls the J index, which appears to be the mean cumulative departure, resulting from the fact that the true cycle is a fractional period.

It is not difficult to detect a periodic quality in data on rainfall and other natural phenomena. For any given span of data, the most suitable period found in a first analysis will inevitably result in some improvement of estimate over that based on the arithmetic mean of experience, as defined by the standard deviation or probable error. Johnson says, in his introduction, that an analysis based on a 20-year period for the rainfall of the Kualapuu, Molokai, area gave "very close agreement, estimated roughly in practical value as 88 %, if fulfilled in the future.

[1] Geologist, Honolulu Board of Water Supply.
Manuscript received March 25, 1947.

In an effort to solve the remaining 12 %, the writer became entangled in an investigation which led far afield."

Moreover, any other systematic relationship found in random values for a given quantity will, if applied, result in more accurate estimates than the use of simple averages without such additional terms. For example, various stations considered by Nakamura in his study of rainfall variation on Oahu have a probable departure of the rainfall of any given year from the mean of about 30 per cent of that mean (Nakamura, 1933). This figure is true of stations having moderate rainfall, such as that of Kualapuu or U. S. Weather Bureau at Honolulu. This means that, by naming the mean annual rainfall, anyone can predict, on the average, within a range of 30 per cent. One means of improving estimate is offered through a recent study of geographic variation (Wentworth, 1946). In this paper it is shown that if the rainfall of near-by, similar stations for the same year is known, the probable error of estimate is cut down, on Oahu, to about 14 per cent. This relationship is of no use for future prediction, but is offered simply as an example of refinement of estimate due to added knowledge.

Parallel to this, Johnson says his errors were reduced to 12 per cent by use of his data on periods. This seems reasonable, and it is shown below that harmonic analysis of the Honolulu Rainfall Index can furnish a basis for somewhat similar improvement of average estimate for a near-by, future year, over that represented by the simple annual mean or normal.

So far, the studies made by Johnson appear to be significant. Beyond this point, in the search for the remaining 12 per cent, it is less clear either that the continued harmonic analysis of residuals is worth the greatly protracted labor involved, or that similarities of period found between rainfall and other physical phenomena are truly the result of cause and effect. Certainly, if causation is at work and if the records of such correlated sunspots, magnetic values, or planetary positions are of long duration or can be reliably projected into the future, the correlation with rainfall would be of much value.

The writer is not sufficiently acquainted with these other phenomena to judge independently whether the similarities in period do or do not represent a causal relationship. The latter half of Johnson's book constitutes an interesting and exhaustive search for relationships between rainfall and other phenomena and there is abundant evidence of cyclical variations which can be to some extent defined. However, candor compels us to conclude that true causal relationship has not been proved, and it does not appear that use of data from planetary or other changes will yield predictions superior to those based on one or two empirical rainfall periods alone.

It is noteworthy that in a lecture given in Honolulu in 1946, Dr. C. G. Rossby expressed the view that useful predictions for more than 1 month in advance are not now practicable. This eminent authority had just completed a tour of conference and inspection in Hawaii, under the sponsorship of the Hawaiian Sugar Planters' Association and the Pineapple Research Institute, and although he recommended renewed and more extensive study of the problem, he was not optimistic of success by available methods in the foreseeable future.[2] In line with this statement, the paper by Johnson, while it probes interesting possibilities, did not, in the estimate of the present writer, contain anything to show that prediction of the rainfall of a single future year could be made

[2] After this paper was completed, the writer had opportunity to see a manuscript report by H. Landsberg, prepared in collaboration between the U. S. Weather Bureau and the University of Chicago, in which a similar view was expressed.

with useful accuracy. For this reason it was decided to examine the Honolulu Rainfall Index by a similar method for a further check (see Table 1).

TABLE 1

THE HONOLULU INTAKE RAINFALL INDEX*

YEAR IN DECADE	DECADES					
	1890	1900	1910	1920	1930	1940
0	132	92	122	82	108	70
1	85	101	134	108	94	79
2	91	137	85	91	117	109
3	91	110	102	124	79	91
4	100	113	100	81	94	74
5	100	107	106	81	94	64
6	66	98	134	60	109	86
7	61	123	109	154	116	......
8	117	88	115	83	103	......
9	81	95	70	87	117	......

* This index is now called the "intake" index, to distinguish it from a "residential" index used for correlation with domestic consumption on lawn irrigation and the like.

This index is the average of the percentages of the mean for 10 stations in the intake area of the Honolulu water supply. It is much more representative of the rainfall which provides water supply than is the record of the official U. S. Weather Bureau station, located in coastal Honolulu in the low rainfall belt.

Our purpose here is to determine whether the values of the Honolulu index, taken on an annual basis, show systematic cycles in such a way that the rainfall of even 1 or 2 years in advance can be predicted with useful validity. No attempt has been made here to correlate cycles of annual rainfall with any other natural cycles. Obviously, no value can be attached to any method of prediction unless it follows a definite routine which can be carried through uniformly by any two persons who follow the adopted rule.

The first operation, similar to that used by Johnson, and others previously, is to calculate the mean differences between annual rainfall indexes separated by each of the intervals considered as a possible cycle. For example, the differences between the indexes of the pairs 1890 and 1895, 1891 and 1896, 1892 and 1897, carried through to 1940 and 1945, are computed and averaged, without regard to sign. This value represents the average and also the most probable amount by which the rainfall of any given year will differ from one 5 years in the past or 5 years in the future. The same calculations were made for a 4-year interval, for 6 years, and so on, up to 20 years, with results shown in Table 2.

The 4-year difference is the mean of 52 values and that for 20 years is the mean of 36 values. It is evident that the means have only a moderate range, that for a 16-year interval being the lowest, 20.8 per cent. Moreover, there is no marked superiority of this interval over the next, and so on. Thus the cyclical character is not strong.

In order to estimate how much of the variation in annual rainfall is periodic, we first need a measure for random variation. Taking the whole series of 56 years, we find the standard deviation to be 20.3 per cent.

TABLE 2

MEAN VARIATIONS IN THE HONOLULU RAINFALL INDEX FOR CYCLES FROM 4 YEARS TO 20 YEARS

LENGTH OF CYCLE	MEAN DIFFERENCE FOR YEARS ONE INTERVAL APART	RANK, LOWEST TO HIGHEST
Interval	Per cent	
4	22.4	6.0
5	21.6	3.5
6	24.3	12.0
7	22.5	7.5
8	24.0	10.5
9	21.8	5.0
10	26.5	16.0
11	24.9	13.0
12	21.6	3.5
13	24.0	10.5
14	22.5	7.5
15	25.0	14.0
16	20.8	1.0
17	25.9	15.0
18	28.2	17.0
19	23.7	9.0
20	21.2	2.0

From this is derived the probable error of estimate if any given year in the future is predicted to have a rainfall equal to the mean of the whole period.

$$\text{P. E.} = 0.6745\ (20.3) = 13.7 \text{ per cent}$$

This means that the odds are equal, that there is a fifty-fifty likelihood that the rainfall in any future year will not differ from the mean by more than 13.7 per cent. This is a basis of prediction that is sound and easily arrived at. Now we must inquire whether, by use of cycles shown in the record, we can reduce this probable error of estimate or prediction.

Taking the 16-year cycle as most promising, we say that the rainfall of any given year is most likely to be similar to that of a year 16, or 32, or 48 years earlier. For such a group of years, the best representative value is the average, and for each year, the deviation from that group average is the error that would have resulted from using the average. Hence we take the whole series of deviations of the rainfall of each year from the average of the years separated from it by multiples of 16. The root-mean-square of this series is determined. This is the standard deviation of the estimates based on use of the mean of the 16-year cycle. For any single series there are only 4 years, and it is granted that the average can be much distorted by a very exceptional year. However, by including the whole series in the root-mean-square calculation we derive a standard deviation comparable to the standard deviation about a single mean. This turns out to be 14.4 per cent. The corresponding probable error is 9.7 per cent. This suggests that by using the mean of the 16-year cycle, rather than the mean of the whole series, the fifty-fifty likelihood is that the estimate will not be in error by more than 9.7 per cent. Here we have an apparent improvement of about one third of the expected error involved in using the over-all mean. This is an improvement which can be used, even though it is not startling.

The following table shows the standard deviations and probable errors of variations from the over-all mean and from the group means for a few of the best and a few of the worst cycles. In offering these data, the writer makes no claim to having discovered a cycle that will have general value or that will be found useful for any other series than the Honolulu Rainfall Index. For any other series a similar procedure will indicate whether any cycle appears to have at least short-term prediction value.

TABLE 3

MEASURES OF ESTIMATE

CYCLE	STANDARD DEVIATION	PROBABLE ERROR
	Per cent	Per cent
Variations from arithmetic mean of 56 years, no cycle	20.3	13.7
Variations from group mean on 16-year cycle	14.4	9.7
Variations from group mean on 20-year cycle	15.5	10.5
Variations from group mean on 10-year cycle	19.2	12.9
Variations from group mean on 18-year cycle	18.6	12.5

The above values indicate the probable error of any given estimate based on the use of the cycle named, assuming that the cycle average is correct. However, this is an unwarranted assumption since the cycle averages (such as the mean of the years 1890, 1906, 1922, and 1938) are at most based on four values for the 16-year cycle and only three for the 20-year cycle. We therefore must examine the source of these averages. For the 16 groups of years spaced by 16 years, we find that the probable errors of the means range from the maximum of 8.6 per cent to 1.6 per cent, with a mean of 4.95 per cent. For the 20 groups of years spaced by 20 years, these same values are 11.0 per cent, 1.1 per cent, and 5.45 per cent.

Even where the individual group shows a smaller standard deviation, the groups are so small that the measure is not reliable. We have no basis for assuming any mean to be closer than average probable errors for the whole series. In Table 4, as an example, an attempt is made to apply the 16- and 20-year cycles, the most promising two cycles, to the prediction of the index for the years from 1939 to 1948 inclusive.

In this table, the actual index up to date is given in column 2. The indexes predicted by the 16-year cycle alone are given in column 4, and those predicted by the 16-year cycle followed by the 20-year cycle are shown in column 6. Columns 3, 5, and 7 show errors of prediction by use of the mean (column 3), by use of the 16-year cycle (column 5), and by use of the 20-year cycle after the 16-year (column 7). These show that, on the average, there is a moderate improvement of prospective accuracy of estimate by using the 16-year cycle, and a still smaller improvement by using the 20-year cycle in addition. However, anyone desiring to use such data for practical purposes should have his doubts aroused by noting that in two of the predicted years, 1946 and 1947, the predicted values in column 4 are quite different from those in column 6 and of opposite deviation from normal. In both 1946 and 1947, there is marked change on using the 20-year cycle. It is easy to trace this effect, since these dates are 20 years after the phenomenally low and high years of 1926 and 1927. The indexes for these 2 years are so aberrant that in much statistical procedure they would be rejected. The 20-year average is based only on two pairs—1906 and 1926, 1907 and 1927—and the effect of the latter year in each case is disproportionate. The figures for 1947 and 1948 are not offered as predictions but simply as working data.

The conclusion drawn from the table and other data presented is that by progressive use of cycles, the prospective average errors of estimation can be made slightly smaller than those shown by predicting the mean.

However, the practical utility of any such improvement is marred by the knowledge that it is gained through the introduction of group averages having probable errors that we cannot reasonably assume to be less than approximately 5 per cent. The problem comes down to whether an estimate having

TABLE 4

PREDICTION BY 16-YEAR AND 20-YEAR CYCLES

1	2	3	4	5	6	7
YEAR	ACTUAL INDEX	DEVIATION FROM MEAN	INDEX BY 16-YEAR CYCLE	DEVIATION FROM ACTUAL	INDEX BY 16- AND 20-YEAR CYCLES	DEVIATION FROM ACTUAL
1939	117	17	112	5	102	15
1940	70	30	83	13	65	5
1941	79	21	87	8	85	6
1942	109	9	98	11	104	5
1943	91	9	120	29	119	28
1944	74	26	77	3	79	5
1945	64	36	79	15	71	7
1946	86	14	108	22	85	1
1947	......	......	94	......	116	......
1948	......	......	114	......	119	......
Average deviation, 8 years	......	20.2	......	12.8	......	9

a probable deviation of 13.7 per cent from a mean of probable error of 1.8 per cent is significantly improved by using an estimate having a probable deviation of 9.7 per cent from a mean of probable error of not less than 5 per cent. Statistical treatment would probably show a slight advantage with the latter procedure in the long run, for the simple reason that it is based on more complete use of the data. But for practical purposes any statistical, long-run gain is cancelled by the evident risk of an aberrant estimate for a given year, as shown in Table 4. If numbers of simultaneous checks for estimates were available, such aberrant estimates might be faced, but predictions of annual rainfall are checked one at a time, with an unavoidable concentration of interest on the one result.

With the present span of data, the averages of rainfall cycles are so subject to disturbance by aberrant years that any given year's prediction can be in error by 20 per cent or more.

It appears that there is at least a psychological difference between estimate and prediction. For every practical purpose we are forced to the conclusion that the cycle analysis does not yield predictions of useful accuracy or reliability. To offer for a given future year, on the present basis, a definite prediction for public guidance in practical matters would be an unwarranted presumption.

REFERENCES

ALTER, DINSMORE. A simple form of periodogram. *Ann. Math. Statis.* 8: 121–126, 1937.

BOARD OF WATER SUPPLY, Honolulu, Hawaii, *Bien. Rpt.* 11. 180 p. Honolulu, 1947.

JOHNSON, MAXWELL O. *Correlation of cycles in weather, solar activity, geomagnetic values and planetary configurations.* viii + 149 p. Phillips and Van Orden, San Francisco, 1946.

NAKAMURA, WINTERS T. A study of the variation in annual rainfall of Oahu island, based on the law of probabilities. *U. S. Monthly Weather Rev.* 41: 354–360, 1933.

WENTWORTH, CHESTER K. *Geographic variation in annual rainfall on Oahu.* 14 p., 4 fig. Hawaii Univ. Res. Pub. 22. Honolulu, 1946.